

**Xavier University of Louisiana  
Math Fair Spring 2024**

**Math Competition (Grade 9-10)**

**Directions:** The use of calculator is **not** permitted. Choose the answer for each of the following problem, circling the correct answer on this page and filling in the appropriate space on the scantron. Be careful and clear. **If none of the given choices is correct, then choose (E) for your answer on the scantron.**

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1. Which of the following is an odd number?

- (A)  $1^4 + 1$       (B)  $3^4 + 2$       (C)  $5^4 + 3$       (D)  $7^4 + 5$       (E)  $1^4 + 7$

2. A box contains red and blue pencils only. If the number of red pencils is two-thirds the number of blue pencils, then the proportion of pencils in the box that are red is:

- (A) 1 : 3      (B) 2 : 3      (C) 1 : 2      (D) 2 : 5      (E) 3 : 5

3. Simplify

$$\left(1 + \frac{1}{2}\right)\left(1 + \frac{1}{3}\right)\left(1 + \frac{1}{4}\right)\left(1 + \frac{1}{5}\right)$$

- (A)  $\frac{1}{120}$       (B)  $1\frac{1}{120}$       (C) 3      (D)  $4\frac{1}{30}$       (E)  $\frac{1}{30}$

4. Evaluate  $(10^{2024})^2 - (10^{2024} - 1)^2$ .

- (A)  $2(10^{2024}) - 1$       (B)  $2(10^{2024})$       (C)  $(10^{2024})^2 + 10^{2024}$       (D) 1

5. If  $\frac{a}{d+b+c} = \frac{4}{3}$  and  $\frac{a}{b+c} = \frac{3}{5}$ , then the value of  $\frac{d}{a}$  is:

- (A)  $\frac{7}{6}$       (B)  $\frac{6}{7}$       (C)  $-\frac{12}{11}$       (D)  $-\frac{11}{12}$       (E)  $\frac{15}{11}$

6. Simplify the following expression

$$\frac{x^3 - 8}{x^3 + 1} \cdot \frac{x^2 - 1}{x^2 - 4x + 4} \div \frac{x^2 - 3x + 2}{x^2 - x + 1}$$

- (A)  $\frac{x^2 - 3x - 7}{x^2 - 5x + 2}$       (B)  $\frac{x^2 + 2x + 4}{(x - 2)^2}$   
(C)  $\frac{(x^2 + 2x + 4)(x - 1)}{(x - 2)(x^2 - x - 1)}$       (D)  $\frac{(x - 2)^3(x - 1)^2(x^2 + 2x + 4)}{(x + 1)^3(x - 2)^2(x^2 - x + 1)}$

7. A mixing bowl is hemispherical in shape, with a radius of 12 inches. If it contains water to half its depth, then the angle through which it must be tilted before the water will begin to pour out is:

- (A)  $15^\circ$       (B)  $30^\circ$       (C)  $45^\circ$       (D)  $60^\circ$       (E)  $75^\circ$

8. A circle and a square have the same perimeter. Then:

- (A) their area are equal  
(B) the area of the circle is the greater  
(C) the area of the square if the greater  
(D) the area of the circle is  $\pi$  times the area of the square

9. The negation of the statement "all dogs are obedient," is :

- (A) no dogs are obedient
- (B) all dogs are disobedient
- (C) some dog are disobedient
- (D) no dogs are disobedient
- (E) some dog are obedient

10. Find an equivalent expression of

$$\frac{\sqrt{2}}{\sqrt{2} + \sqrt{3} - \sqrt{5}}$$

- (A)  $\frac{3 + \sqrt{6} + \sqrt{15}}{6}$       (B)  $\frac{\sqrt{6} - 2 + \sqrt{10}}{6}$       (C)  $\frac{2 + \sqrt{6} + \sqrt{10}}{10}$   
(D)  $\frac{2 + \sqrt{6} - \sqrt{10}}{6}$       (E) None of these

11. Solve for  $x$  from the equation  $x + 1 = \sqrt{22 - 2x}$ .

- (A) 7      (B)  $-7$  or 3      (C) 3      (D)  $1 \pm \sqrt{22}$

12. Let  $i = \sqrt{-1}$  be the imaginary unit. Find the product  $(2 + 3i) \cdot (2 - 4i)$ .

- (A)  $-8 - 2i$       (B)  $16 - 2i$       (C)  $16 + 14i$       (D)  $16 - 14i$

13. Circle  $C_1$  passes through the center of, and is tangent to, circle  $C_2$ . The area of circle  $C_1$  is 4. Then the area of circle  $C_2$  is:

- (A) 8      (B)  $8\sqrt{2}$       (C)  $8\pi$       (D) 16      (E)  $16\sqrt{2}$

14. The price of an object was increased  $p\%$ . Later the new price was decreased  $p\%$ . If the last price was one dollar, the original price, in dollars, was:

- (A)  $\frac{1 - p^2}{200}$       (B)  $\frac{\sqrt{1 - p^2}}{100}$       (C) 1      (D)  $1 - \frac{p^2}{10000 - p^2}$       (E)  $\frac{10000}{10000 - p^2}$

15. Which of the following is true about the equation

$$x^3 - 1 = (x - 1)(x^2 - x + 1)?$$

- (A) It has exactly one real solution
- (B) It has exactly two real solutions
- (C) It has exactly three real solutions
- (D) It has infinitely many solutions, but it is not an identity
- (E) It is true for any value of  $x$

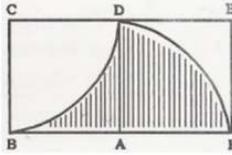
16. Given that  $0 < a < b < c < d$ , which of the following is the largest?

- (A)  $\frac{a + b}{c + d}$       (B)  $\frac{a + d}{b + c}$       (C)  $\frac{b + c}{a + d}$       (D)  $\frac{b + d}{a + c}$       (E)  $\frac{c + d}{a + b}$

17. For an integer  $n$ , what does  $(-1)^{(n^4 - 3n + 2)}$  simplify to?

- (A) 1      (B)  $-1$       (C)  $(-1)^n$       (D)  $(-1)^{n^2}$

18.  $ABCD$  and  $ADEF$  are squares with a common side  $AD$  of length 12 inches. Arc  $BD$  and arc  $DF$  are quarter-circles. How many square inches are in the area of the shaded region?



- (A) 100                      (B)  $121\pi$                       (C) 144                      (D)  $125\pi$                       (E)  $144\pi$
19. The graph of  $|x| + |y| = 4$  is a
- (A) circle                      (B) square                      (C) triangle                      (D) pentagon
20. Suppose  $x$  and  $y$  are non-negative real numbers such that  $x^2 + y^2 = 10$  and  $xy = 7$ . Determine  $x + y$ .
- (A)  $\sqrt{2}$                       (B)  $6\sqrt{2}$                       (C)  $2\sqrt{6}$                       (D) 2                      (E)  $\sqrt{6}$
21. If four mice can eat four pounds of cheese in four minutes, how long will it take 99 mice to eat 99 pounds of cheese?
- (A) 20 min                      (B) 22 min                      (C) 3 min                      (D) 4 min                      (E) 99 min
22. If  $\frac{a + 3b}{a - 3b} = 7$  then what is  $\frac{a + 2b}{a - 2b}$ ?
- (A) 1                      (B) 3                      (C) 5                      (D) 7
23. Find the length of the diameter of a circle whose area is tripled when the length of its radius is increased by 2.
- (A)  $1 + \sqrt{2}$                       (B)  $2 + 2\sqrt{2}$                       (C)  $1 + \sqrt{3}$                       (D)  $2 + 2\sqrt{3}$                       (E)  $2 + \sqrt{3}$
24. **Tie Breaker:** This is graded only to determine the first, second, or third place tie. Show your work and the answer must be an exact numerical value.

In a fraction, its denominator exceeds its numerator by 3542. The fraction expressed in lowest terms is  $\frac{4}{11}$ . What was the original fraction before simplification?

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1. (B)

2.  $R = \frac{2}{3} B$        $\frac{2}{3} B + B = \frac{5}{3} B$

(D)      2 : 5

3.  $\frac{\cancel{2}}{2} \cdot \frac{\cancel{4}}{3} \cdot \frac{\cancel{5}}{4} \cdot \frac{6}{5} = 3$

(C)

4.  $(10^{2024})^2 - (1024 - 1)^2$

(A)

$$= (10^{\cancel{2024}} - 10^{\cancel{2024}} + 1)$$

$$(10^{2024} + 10^{2024} - 1)$$

$$= 2 \cdot 10^{2024} - 1$$

5.  $\frac{a}{d+b+c} = \frac{4}{3}$        $\frac{a}{b+c} = \frac{3}{5}$

(D)

$$\frac{d+b+c}{a} = \frac{3}{4} \quad \frac{b+c}{a} = \frac{5}{3}$$

$$\frac{d}{a} + \frac{b+c}{a} = \frac{3}{4} \quad \frac{d}{a} + \frac{5}{3} = \frac{3}{4}$$

$$\frac{d}{a} = \frac{3}{4} - \frac{5}{3} = \frac{9-20}{12} = -\frac{11}{12}$$


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6.  $\frac{x^3 - 8}{x^3 + 1} \cdot \frac{x^2 - 1}{x^2 - 4x + 4} \cdot \frac{x^2 - x + 1}{x^2 - 3x + 2}$

(B)

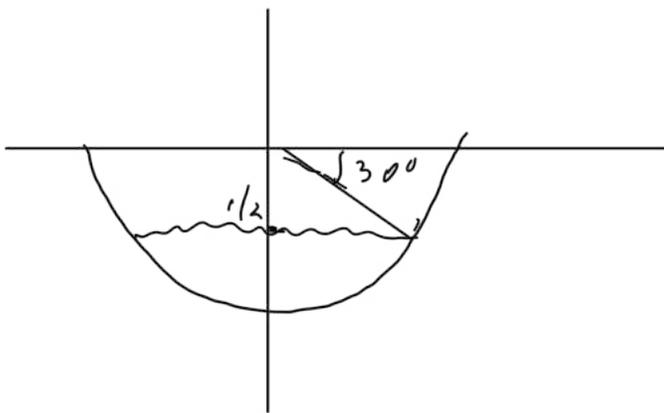
$$\frac{\cancel{(x-2)}(x^2 + 2x + 4)}{\cancel{(x+1)}(x^2 - x + 1)} \cdot \frac{\cancel{(x-1)}(x+1)}{(x-2)^2} \cdot \frac{\cancel{(x^2 - x + 1)}}{\cancel{(x-2)}(x-1)}$$

$$\frac{x^2 + 2x + 4}{(x-2)^2}$$


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7.

(B)



8.

(B)

$$2\pi r = 4x \quad r = \frac{4}{2\pi} x = \frac{2}{\pi} x$$

$$\pi r^2 = \pi \frac{4}{\pi^2} x^2 = \frac{4}{\pi} x^2$$

$$A_{\text{circle}} = \frac{4}{\pi^2} A_{\text{square}} < A_{\text{square}}$$

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9. (C)

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$$\begin{aligned} 10. \quad (A) \quad \frac{\sqrt{2}}{\sqrt{2} + \sqrt{3} - \sqrt{5}} &= \frac{\sqrt{2}(\sqrt{2} + \sqrt{3} + \sqrt{5})}{(\sqrt{2} + \sqrt{3} - \sqrt{5})(\sqrt{2} + \sqrt{3} + \sqrt{5})} \\ &= \frac{2 + \sqrt{6} + \sqrt{10}}{(\sqrt{2} + \sqrt{3})^2 - 5} = \frac{2 + \sqrt{6} + \sqrt{10}}{2 + 3 + 2\sqrt{6} - 5} \\ &= \frac{2 + \sqrt{6} + \sqrt{10}}{2\sqrt{6}} = \frac{2\sqrt{6} + 6 + \sqrt{60}}{12} \\ &= \frac{2\sqrt{6} + 6 + 2\sqrt{15}}{12} \\ &= \frac{\sqrt{6} + 3 + \sqrt{15}}{6} \end{aligned}$$

11.  $x + 1 = \sqrt{22 - 2x}$

(C)

$$\cancel{x^2} + \cancel{2x} + 1 = 22 - \cancel{2x}$$

$$x^2 + 4x - 21 = 0$$

$$(x + 7)(x - 3) = 0$$

$$\cancel{x = -7} \quad \boxed{x = 3}$$

12.  $(2 + 3i)(2 - 4i)$

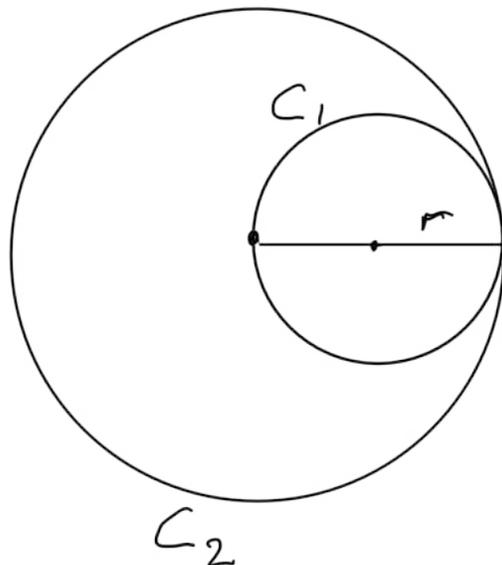
(B)

$$4 + 12 - 8i + 6i$$

$$16 - 2i$$

13.

(D)



$$A_1 = \pi r^2 = 4$$

$$\begin{aligned} A_2 &= \pi(2r)^2 \\ &= 4\pi r^2 \\ &= 4 \cdot 4 = 16 \end{aligned}$$

14.

(E)

$$\begin{aligned}
 X_1 &= X_0 + \frac{p}{100} X_0 \\
 &= \frac{100+p}{100} X_0
 \end{aligned}$$

$$\begin{aligned}
 X_2 &= X_1 - \frac{p}{100} X_1 \\
 &= \frac{100+p}{100} X_0 - \frac{p}{100} \frac{100+p}{100} X_0 \\
 &= \frac{100+p}{100} X_0 - \frac{100p+p^2}{10000} X_0 \\
 &= \frac{10000 + \cancel{100p} - \cancel{100p} - p^2}{10000} X_0 \\
 &= \frac{10000 - p^2}{10000} X_0 = 1
 \end{aligned}$$

$$X_0 = \frac{10000}{10000 - p^2}$$

$$15. \quad x^3 - 1 = (x - 1)(x^2 - x + 1)$$

(B)

$$(x - 1)(x^2 + x + 1) = (x - 1)(x^2 - x + 1)$$

$$(x - 1)(\cancel{x^2} + x + \cancel{1} - \cancel{x^2} + x - \cancel{1})$$

$$(x - 1)2x = 0$$

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16. (E)

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$$17. \quad (-1)^{n^4 - 3n + 2} = (-1)^{n^4 - n} = (-1)^{n - n} = 1$$

(A)

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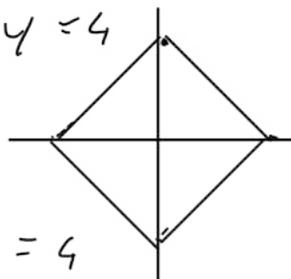
$$18. \quad 12^2 = 144$$

(C)

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19.

$$-x + y = 4$$



$$-x - y = 4$$

$$y = -x - 4$$

$$x - y = 4$$

$$y = x - 4$$

(B)

20.  $x^2 + y^2 = 10$        $xy = 7$

$2xy = 14$

(C)

$$x^2 + 2xy + y^2$$

$$= 10 + 14 = 24$$

$$(x+y)^2 = 24$$

$$x+y = \sqrt{24} = 2\sqrt{6}$$

21.  $C = \#$  of lbs eaten in 1 min by 1 mouse

(D)

$4C = \#$  of lbs eaten by 4 mice in 1 m.

$16C = \#$  of lbs eaten by 4 m. in 4 min.

$$16C = 4 \quad C = \frac{1}{4}$$

$T =$  time to eat 1 lb. by 1 m.

$$T = 4$$

4-99 = time to eat 99 lbs. by 1 m.

4 = time to eat 99 lbs by 99 m.

22.

(B)

$$\frac{a + 3b}{a - 3b} = 7 \quad x = \frac{a}{b}$$

$$\begin{aligned} \frac{x + 3}{x - 3} &= 7 & x + 3 &= 7x - 21 \\ & & 6x &= 24 \\ & & x &= 4 \end{aligned}$$

$$\frac{x + 2}{x - 2} = \frac{6}{2} = 3$$

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23.

(D)

$$A_0 = \pi r^2$$

$$\pi(r + 2)^2 = 3\pi r^2$$

$$r^2 + 4r + 4 = 3r^2$$

$$2r^2 - 4r - 4 = 0$$

$$r^2 - 2r - 2 = 0$$

$$r = \frac{1 + \sqrt{1+2}}{1} = 1 + \sqrt{3}$$

$$d = 2 + 2\sqrt{3}$$

$$24. \quad \frac{N}{N+3542} = \frac{4}{11}$$

$$11N = 4N + 2^3 \cdot 7 \cdot 11 \cdot 23$$

$$3542 = 2 \cdot 7 \cdot 11 \cdot 23$$

$$7N = 2^3 \cdot 7 \cdot 11 \cdot 23$$

$$N = 2^3 \cdot 11 \cdot 23 = 2024$$

$$\begin{array}{r} 2024 \\ \hline 5566 \end{array}$$